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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 685

CIRCULATION MEASUREMENTS ABOUT THE TIP OF AN AIRFOIL DURING FLIGHT THROUGH A GUST

By Arnold M. Kuethe
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By Arnold M. Kuethe

SUMMARY

Measurements were made of the circulation about the rectangular tip of a short-span airfoil passing through an artificial gust of known velocity gradient. A Clark Y airfoil of 30-centimeter chord was mounted on a whirling arm and moved at a velocity of 29 meters per second over a vertical gust with a velocity of nearly 7 meters per second. Flow angles were measured with a hot-wire apparatus.

The rate at which the lift at the tips of a wing entering a gust is realized was found to be in satisfactory agreement with that predicted on the basis of the two-dimensional theory of von Kármán and Sears.

INTRODUCTION

The problem of the unstationary flow about wings has received considerable attention in recent years because of its wide application, particularly to the problems of flutter and the forces experienced by airplanes during unsteady flight. The specific problem of the unstationary two-dimensional flow about an airfoil during flight through gusts has been investigated theoretically by Küssner (reference 1) and by von Kármán and Sears (reference 2). To date there have been no experimental investigations substantiating these theories on the important point of the rate at which the lift of a wing is realized during its penetration into a gust. Farren's measurements (reference 3) apply to the similar case of the rate of change of the circulation about an infinite wing started impulsively from rest. The results agree very well with the corresponding theoretical results by Wagner (reference 4). This case is similar to the problem of the passage of an airfoil into a gust with the important simplification, however, that the entire chord of the airfoil is assumed to strike the gust at the same instant.

The three-dimensional problem of the penetration of an airfoil into a gust presents extreme theoretical difficulties. The problem is obviously important from considerations of the gust loads at the wing tips and on the control surfaces. Küssner (reference 1) points out that, in the first moment, the lift distribution near the tips of a wing should be practically the same as that predicted theoretically if the induction due to the trailing vortices is neglected. The justification for this statement lies in the fact that, near the gust entrance, the trailing vortices extend only a short distance behind the wing. Since in steady flow, the velocities induced by the trailing vortices cause the lift near the tip of a wing to drop off markedly from that predicted by the two-dimensional theory, the foregoing statement indicates the possibility of tip loads in gusts exceeding those predicted on the basis of steady-flow tests.

The present investigation throws light on this important problem. The results of measurements of the circulation about the rectangular tip of a short-span airfoil during flight through an artificial gust of known velocity gradient are given. The work was carried out at the Daniel Guggenheim Airship Institute with the financial assistance of the National Advisory Committee for Aeronautics.

The author wishes to express his appreciation to Dr. Th. Troller for valuable suggestions during the course of the work; to M. E. Long, who carried out the preliminary measurements; and to M. W. Seese, who worked up most of the final data and curves.

METHOD AND APPARATUS

Method

The method by which the flow about the tip of an airfoil was investigated is illustrated in figure 1. The airfoil was mounted on the whirling arm at the Daniel Guggenheim Airship Institute and, in the path of the model, a vertical jet was maintained. Hot-wire instruments measuring the speed and the direction of the flow were placed successively at many points in the wake of the airfoil. Continuous oscillograph records of the air speed and direction served to define a rectangle ABCD (fig. 1) that enclosed the tip trailing vortices throughout the motion of the airfoil through the vertical jet. After this rectangle was established, a final set of measurements in which the hot-wire instruments were placed successively at various points on the boundary of this rectangle permitted the plotting of "additional velocity" (cross- and vertical-velocity) diagrams with distance along the boundary as the abscissa. The area under these diagrams then gave the circulation of the vortices enclosed by the rectangle. Several "additional velocity" diagrams corresponding to the various stages of the penetration of the airfoil into the gust were integrated to find the circulation about the tip as a function of the time. The plane of the measuring rectangle ABCD was about 8 centimeters behind the airfoil trailing edge.

Airfoil and Gust Apparatus

The airfoil used in the investigation is of Clark Y section with square tips; it has a chord of 30 centimeters and an aspect ratio of 3. In order to eliminate the turbulence generated at the sharp edges and its disturbing effect on the flow measurements, a rounded plasticine form was fitted to the tip. Figure 2 shows the airfoil with its rounded tip mounted on the whirling arm. The hot-wire mounting is shown projecting to the rear.

A rectangular airfoil was chosen for the reason that any effects due to the limited extent of the trailing vortices behind the airfoil in the early stages of the motion would be more noticeable for this plan form than for, say, a tapered wing, which shows a relatively smaller reduction in lift at the tips due to the induction of the trailing vortices.

The investigation was carried out by means of the whirling arm and the gust tunnel at the Daniel Guggenheim Airship Institute (reference 5). The wing model was mounted on the arm and, in the path of the model, a vertical current of air was maintained. Sectional views of the whirling arm and the gust tunnel are shown in figure 3. The arm is 10 meters long; model velocities up to 75 meters per second at 75 r.p.m. can be obtained. The model moves in a circular tunnel of rectangular cross section, 5.0 meters wide by 6.2 meters high.

The vertical current of air is the free jet of a continuous-circuit wind tunnel. The open jet is in the path of the model and measures 4.9 meters across by 4.57 meters in the direction of movement. The jet is produced by a four-speed induction motor driving a two-blade propeller. The wind velocity is variable in four steps from 4 to 12 meters per second.

The gust profile used for the tests is shown in figure 4. It shows a rapid change of vertical velocity from 0 to 5.2 meters per second in 0.45 meter followed by a gradual increase to 6.7 meters per second over the following 0.87 meter. For the preliminary measurements described later, a less sharp gust was used. In this case the rapid increase in vertical velocity took place over about 0.7 meter; otherwise, the profile was nearly identical with that shown in figure 4.

The measurements were carried out at a model velocity of 29 meters per second (30 r.p.m. of the arm). The Reynolds Number was accordingly about 600,000. The airfoil was set at 0° with respect to its direction of movement. Then the angle of attack reached a maximum of $\tan^{-1}(6.7/29) = 13°$ in the gust.

Hot-Wire Apparatus

The flow angles were measured by a hot-wire arrangement, the essential features of which are shown in figure 5. The principle of the instrument was developed at this laboratory by F. D. Knoblock (reference 6). A potentiometer circuit for measuring the current through and the potential drop across the hot wire is not shown. A fine platinum wire 0.014 millimeter in diameter was stretched between and soldered to three prongs so that the two branches were of the same length and the subtended angle was about 80°. Figure 6a is a photograph of the instrument; each hot-wire branch was about 7 millimeters in length. The two branches of the hot wire were in adjacent arms of a Wheatstone bridge circuit, and the unbalance of this circuit was shown by the deflection of the oscillograph element. The bridge was balanced when the direction of the wind bisected the angle between the two branches of the hot wire. If the direction changed to a (fig. 5) the upper wire was cooled more than the lower, and the resulting unbalance of the bridge was shown by the oscillograph deflection. A continuous photographic record of the oscillograph deflection then gave, after reference to a suitable calibration chart, the direction of the air flow as a function of the time. An auxiliary circuit was so arranged that the oscillograph was short-circuited shortly before entering and shortly after leaving the gust. These marks on the film made it possible for the edges of the jet to be located on the records and gave a "zero line" from which deflections were measured. Figure 7 is a reproduction of one of the oscillograph records. The zero marks and a 60-cycle timing wave, by means of which the jet boundaries were placed on the records, are also shown.

The jet boundaries were taken at the points directly above the edges of the inlet section. When the instrument was mounted so that the three prongs were in a vertical plane, the oscillograph deflection was a measure of the pitch of the flow; when the prongs were in a horizontal plane, the deflection was a measure of the yaw.

The direction unit had a sensitivity of about 1 millimeter oscillograph deflection per degree deviation of the flow. Calibrations carried out over the range of velocities encountered during the tests (27 to 32 meters per second) showed no noticeable effect of the velocity on the sensitivity. The interdependence between pitch and yaw was, however, appreciable. For instance, if the flow was inclined both in pitch and yaw, both of the angles indicated were in error depending on the magnitude of the other. This error amounted to about 11 percent for a deviation of 10° in the direction perpendicular to the component being measured. This source of error was minimized by measuring both pitch and yaw at each position and applying the predetermined correction factor.

Calibrations were carried out by means of a small wind tunnel. Calibrations of a given hot-wire unit were carried out every 3 or 4 hours during the course of the measurements.

The hot-wire instrument described gives the direction of the flow <u>only</u> if the velocity is the same over the two branches of the wire. In other words if, say, the velocity over one branch was higher than it was over the other, a differential cooling of the two branches took place and an angle was indicated even though the flow was not inclined. By means of the hot-wire unit shown in figure 6b, it was possible to ascertain whether, at any particular position, the velocity over the length of the wire was sufficiently uniform so that the direction records could be relied upon. This unit was similar to the instrument of figure 6a except that the angle between the two branches was 180°. It was, accordingly, insensitive to direction changes but sensitive to velocity differences between the upper and the lower branches. The sensitivity to the velocity difference between the branches was adjusted so that it had the same value as for the pitch instrument. When this unit was mounted at positions behind the airfoil, it was possible to determine the approximate error in the indicated pitch or yaw at the same position due to velocity difference between the two branches of the direction unit.

This procedure was necessary because the turbulence and the strong velocity gradients at some positions on the wake invalidated direction records taken at those positions. At most of the positions finally used, the oscillograph deflection indicated zero velocity difference, and the average error around a closed circuit was a small fraction of a degree in direction, or within a tenth of a meter per second in the calculated cross or vertical velocity. The corresponding error in the circulation is not more than a few percent of the maximum value.

Velocities in the wake behind the airfoil were measured by a single hot wire in one of the branches of the Wheatstone bridge shown in figure 5, replacing one branch of the direction unit. The other branch of the direction unit was replaced by a constant resistance, the value of which was adjusted so that the oscillograph deflection was near zero for the mean wind velocity. Records of wind velocity were taken at several points behind the airfoil in order to determine the velocity to be used in calculating the cross and the vertical velocities from the measured flow directions. Records given later show a negligible variation in velocity about the closed circuits investigated.

A source of error near the jet boundaries is that due to the lag of the hot wire when its temperature is changing rapidly. The lag of the wire is determined by its time constant (reference 7), which can be determined for any given set of conditions. A graphical construction, given in reference 7, was utilized to correct the records for this lag.

Accuracy

The sources of error introduced by the hot-wire apparatus have been described in the preceding section.

Turbulence caused some deviation between successive records at the same position, especially near the entrance of the gust. For this reason, all experimental points given in the following section represent the mean value of two passages through the gust.

On the basis of the sources of error described, it is concluded that the circulation measurements given are correct to within 8 to 10 percent of the maximum circulation.

RESULTS

A preliminary set of measurements, consisting of about 180 records of velocity, pitch, yaw, and velocity difference was taken in a plane 8 centimeters behind the trailing edge of the rectangular airfoil during its passage through the diffuse gust mentioned under Method. The velocity of the model was 29 meters per second. The object of these measurements was to determine the position of the trailing tip vortices in the plane behind the airfoil. This determination was necessary in order to fix the circuit along which measurements of the cross and the vertical velocities would lead to a determination of the strength of all the trailing tip vortices. Pitch and yaw traverses along various lines, in a plane perpendicular to the direction of motion 8 centimeters behind the trailing edge, at intervals of 0.25 meter along the path of the airfoil near the gust entrance, are shown in figure 8. The separate diagrams represent the progressive movement of the airfoil into the gust. The time interval between separate diagrams was $0.25/29 = 0.0086$ second. In these diagrams, a is the distance above the lower surface in centimeters, b the distance behind the airfoil trailing edge in centimeters, and c is the distance out from the tip in centimeters. The expression $x = 0$ signifies that the hot wire is directly above the edge of the jet mouth. The leading edge of the airfoil strikes the gust transition zone at $x = -0.45$ meter. For these preliminary measurements, the records are uncorrected for the lag of the hot wire, or for the angle of the flow in the perpendicular plane.

The angle profiles of figure 8 show that the center of the tip trailing vortex was at $a = 0$, $c = -2.5$ centimeters in the undisturbed flow ($x = -0.50$ meter) and at $a = 3.5$ centimeters, $c = -2$ centimeters when the airfoil was completely inside the gust ($x = 1.75$ meters). It was concluded from these diagrams that practically all of the vorticity shed from the tip was contained in the area bounded by the lines $a = -4$, $a = 10$, $c = -5$, $c = 6$ centimeters. The measurements were not sufficiently comprehensive to allow a reliable determination of the circulation about this rectangle or about one with these approximate dimensions.

On the basis of the vorticity field defined by the preliminary records, a final set of measurements, consisting of 133 records of velocity, pitch, and yaw, was taken at many positions along the lines $a = -4$ and 10 centimeters, $c = -5, -7$, and 6 centimeters.

Velocity records taken at various positions on these lines, plotted against distance from the hot wire to the jet entrance in meters, are shown in figure 9. On the basis of these measurements, it was concluded that a constant velocity of 29 meters per second could be assumed for the calculation of the cross and the vertical velocities.

All the direction records of this final set of measurements were corrected for the lag of the hot wire and for the angle of the flow in the perpendicular plane. The angles of the flow were read off the corrected curves at x intervals of 0.25 meter over a distance of 3 meters, from $x = -1$ to $x = 2$ meters. The curves were analyzed only near the gust entrance because the analysis is laborious and the results obtained at the exit could only be a repetition of those at the entrance. The values of the flow angles about the rectangles in the wake were then used to calculate the instantaneous values of the circulations for the various stages of the motion.

The curves of additional velocity w about the rectangles ABCD and AB_1C_1D in the wake at intervals of 0.25 meter near the gust entrance are shown in figures 10a and 10b, respectively. The dimensions of the rectangles are given in figure 10a. The ordinate along AB is $U \sin \alpha_y$ where U is the mean velocity (29 meters per second) and α_y is the corrected yaw angle (positive inward) at several positions along the line. Along BC and B_1C_1 the ordinate is $-U \sin \alpha_p$ where α_p is the corrected pitch angle (positive upward). Along CD the ordinate is $-U \sin \alpha_y$ and along DA, $U \sin \alpha_p$.

The curves of figure 10 were integrated to find the circulation of the tip trailing vortices as a function of the penetration of the airfoil into the gust. These curves are shown in figure 11 for two rectangles ABCD and AB_1C_1D. A scale of C_L based on the formula $C_L = 2\Gamma/(U \times \text{chord})$ and a scale of chords, on which zero corresponds to the point where the leading edge of the airfoil strikes the gust entrance, are also shown.

Before these results are considered further, the theoretical curves based on the two-dimensional theory will be derived. Von Kármán and Sears (reference 2) considered the case of an airfoil penetrating an infinitely sharp gust. They obtained a curve of lift against penetration of the airfoil into the gust. The lift comprises two components, that due to the circulation and that due to the apparent mass; the measurements reported here lead to an evaluation only of that part due to the circulation. According to von Kármán and Sears, the contribution due to the apparent mass* is

$$2 \rho U W \sqrt{2 U t - U^2 t^2}$$

where U is the velocity of the airfoil through the discontinuity in half chords per second, W is the gust velocity or the height of the discontinuity in the velocity normal to the airfoil, and t is the time. The value of $t = 0$ corresponds to the instant when the leading edge enters the discontinuity. The foregoing expression must be deducted from the total lift to obtain that due to the circulation. It will be observed that the apparent mass contribution is real only in the range $0 < t < 2$, i.e., while the airfoil spans the discontinuity. For $t > 2$, i.e., after the airfoil is completely within the gust, the contribution due to the apparent mass is zero. The curve marked $\Gamma_1(t)/\Gamma_\infty$ in figure 13 represents the circulation about the airfoil for a unit sharp gust as obtained by deducting the foregoing apparent-mass contribution from the von Kármán - Sears lift curve.

The circulation as a function of the time for the gust profile used in the present investigation was derived by the method of superposition suggested by Jones (reference 8). This method utilizes the fact that any gust is built up by a superposition of an infinite number of discrete steps, for each one of which the law governing the rate of change of the lift due to an infinitely sharp gust applies.

The theorem, applied to the present problem, states that: if $\Gamma_1(t)$ is the circulation function for a unit infinitely sharp gust and $\Gamma(t)$ is the circulation function for a gust $W(t)$ then

*This expression results from a differentiation with respect to the time of equation (48) in the von Kármán-Sears paper.

$$\Gamma(t_0) = \Gamma_1(t_0)\, W(0) + \int_0^t \Gamma_1(t_0-t)\, W'(t)\, dt$$

where $\Gamma(t_0-t)$ is the circulation at a time (t_0-t) after the impression of the unit gust. In the present case, $W(0) = 0$ and the equation can be put in the convenient form

$$\Gamma(t) = \int_0^{W(t_0)} \Gamma_1(t_0-t)\, d\, W(t)$$

in which form it can be easily solved by the graphical method given by Jones. The curve marked $W(t)$ in figure 12 is the gust profile of figure 4 plotted to a time scale based on the airfoil velocity of 29 meters per second. For the purposes of this calculation, zero time corresponds to the instant when the leading edge strikes the gust transition zone (at $x = -0.45$ meter). The ordinate is W/W_{max} where $W_{max} = 6.7$ meters per second. The angle of attack of the wing in the gust is $\tan^{-1}(6.7/29) = 13^{\circ}$. The time scales for the $\Gamma_1(t)$ and $W(t)$ curves are the same, but the latter is turned through 90°. Lines from the $\Gamma_1(t)$ curve are projected across and lines from the $W(t)$ curve are projected up. The locus of the intersections, such that the sum of the two time values is constant, gives the curves shown in figure 12 designated by $\Gamma_1(t_0-t)$. The area under a definite curve is $\Gamma(t)$ where t is the sum of the two time values corresponding to the abscissas of the points through which the intersecting lines were drawn. $\Gamma(t)$ is plotted in the same figure with $\Gamma_1(t)$.

In order to compare the measured values with the theoretical curves as given in figure 12, it is necessary to know C_L at that point of the span such that all the trailing vortices included in the rectangles ABCD or AB_1C_1D originate between that point and the tip. The points B and C, and B_1 and C_1 are 5 and 7 centimeters in from the tip, respectively. Then, to carry out the comparison, it is necessary to know C_L at about one-fifth chord from the tip of a wing with an aspect ratio of 3. As far as the author knows, pressure distributions on a rectangular wing of aspect ratio 3 have not been made. Examinations of theoretical pressure distributions (reference 9, fig. 63) indicate that C_L at one-fifth chord in from the tip is of the order of 12 percent below the mean C_L for the wing.

Existing force measurements on Clark Y airfoils in steady flow show that, for an aspect ratio of 3, $C_L = 0.28$ at 0° and $C_L = 1.05$ at 13° angle of attack. The value of 88 percent of the difference between these values, or 0.63, is taken as the increment in the local lift coefficient at one-fifth chord inward from the tip. The corresponding circulation increment is $\frac{1}{2}(0.63 \times 29 \times 0.3) = 2.74$ meters² per second. Calling this value $\Delta\Gamma_\infty$ and the experimentally determined circulation increment $\Delta\Gamma$, $\Delta\Gamma/\Delta\Gamma_\infty$ is directly comparable with the theoretical curve $\Gamma(t)$ of figure 12. These values are plotted together in figure 13.

DISCUSSION

The vertical inclination of the tip trailing vortices behind the airfoil is most clearly shown in the pitch traverses of figure 8. The curves show that, outside the gust, the axes of the tip trailing vortices was at $a = 0$, changing to $a = 3.5$ centimeters inside the gust. The position of the maxima in the W curves along BC, B_1C_1, and DA (fig. 10) indicate that the induced velocities are greatest, and hence that the axes of the tip trailing vortices lie along the line $a = 3$ centimeters, compared with $a = 3.5$ centimeters, as shown in figure 8.

Inasmuch as the measurements were taken in a plane 8 centimeters behind the trailing edge, the preceding values show that the axis of the vortex was inclined upward at a considerable angle. If it is assumed that the vortex originated at, say, the half-chord position, the inclination was $\tan^{-1}\left(\frac{3.2}{15 + 8}\right) = 8°$. The yaw curves of figure 8 show that the mean flow was directed inward from the tip at an angle of about 4° with respect to the airfoil chord.

The curves of figure 10 give further insight into the momentary nature of the trailing vortices as the airfoil penetrates into the gust. If the trailing vortices were concentrated at the tip, one would expect the W curves along AB and CD to have the same character as they have along BC and DA, i.e., they would show well-defined maxima. On the other hand, if the trailing vortices are represented by a sheet of vorticity extending inward from the tip, the W curves along AB and CD would start at a high value inward and end at a low value outward from the tip. Actually, the curves show that the first of these conditions obtains for $x < 0$ and the second obtains for $x > 0$. Since the leading edge of the airfoil strikes the gust transition zone at $x = -45$ centimeters and the chord of the airfoil is 30 centimeters, this behavior indicates that, until the airfoil is well within the transition zone, the trailing vortices are concentrated at the tip, giving way thereafter to a sheet of vorticity extending inward from the tip.

The circulation curves of figures 11 and 13 further support this conclusion. The fact that the circulation curves for the rectangles ABCD and AB_1C_1D practically coincide near the gust entrance, indicating that at the first moment there are no trailing vortices in the rectangle BCC_1B_1, is in agreement with this observation.

The lift coefficients derived from the measured circulations and plotted in figure 11 can be compared with those experienced in steady flight. It should be pointed out that the circulation measured is that corresponding to the local lift at that point on the wing such that all trailing vortices originating at positions nearer the tip pass through the measuring rectangle and all those originating farther from the tip pass outside the rectangle. The lift coefficient derived from the measurements outside the gust (0.15) does not agree well with that predicted on the basis of wind-tunnel tests (0.28) at 0° angle of attack.

The increment in the circulation due to the gust is, however, in good agreement with that predicted on the basis of the wind-tunnel tests and the theoretical rate of increase of the circulation when account is taken of the falling off of the lift near the tip. The process by which this agreement is established is explained in the preceding section and shown in figure 13. The von Kármán-Sears theory of reference 2 predicts that the circulation will build up to 0.83 of its final value at $x = 2$ meters. The experimental points show values of 0.82 and 0.88 for the rectangles AB_1C_1D and ABCD, respectively.

No attempt was made to distinguish between the predicted lift coefficients corresponding to the two rectangles, since the line B_1C_1 is only 2 centimeters nearer the tip than BC.

The circulation curves of figure 13, which represent the increment in the circulation due to the gust, diverge from the theoretical curves based on the von Kármán-Sears calculation but the theoretical and the experimental curves are nearly parallel. Inasmuch as the von Kármán-Sears calculation applies to the two-dimensional case and the experimental results given here apply to a three-dimensional case, exact agreement between the two is not to be expected. The divergence is in the right direction, moreover, in view of the indications pointed out that near the gust entrance the trailing vortices are concentrated at the wing tip. This condition would result in a momentary lift higher than that predicted on the basis of steady wind-tunnel tests. Part of this divergence may also be due to uncertainties in the gust profile of figure 4 near the gust entrance, where velocities are low and the turbulence is extremely high. In view of these factors, the agreement between the theoretical and the experimental results is considered satisfactory.

It is further evident from these curves that momentary values of the lift coefficient near the tips of a rectangular airfoil do not exceed those measured in steady flight. The induction due to the trailing vortices, resulting in a reduction of the lift near the tip, is greater for the rectangular plan form than it is for those plan forms in common use, say, the tapered wing. Hence, since the shorter trailing vortices during the early stages of the penetration into the gust do not result in excessive tip loads in the case of a rectangular airfoil, it is concluded that, for conventional plan forms, wing-tip loads during flight through gusts do not exceed those predicted on the basis of steady-flow tests.

CONCLUSIONS

The conclusions drawn on the basis of the foregoing results and discussed in the preceding section are:

1. During the first stages of the motion of a wing into a gust, the measurements indicate that the trailing vortices behind a wing of rectangular plan form are concentrated nearer to the wing tips than they are during flight.

2. For conventional plan forms, i.e., with the chord

constant or decreasing toward the tips, wing-tip loads in gusts do not exceed those experienced in steady flight at angles of attack equal to the maximum value reached in the gusts.

7. The rate at which the lift at the tips of a wing entering a gust is realized is in satisfactory agreement with that predicted on the basis of the two-dimensional theory of von Kármán and Sears. The deviation between the theoretical and the experimental results shown in figure 13 is due, at least in part, to the momentary character of the flow stated in conclusion 1.

Daniel Guggenheim Airship Institute,
Akron, Ohio, November 1938.

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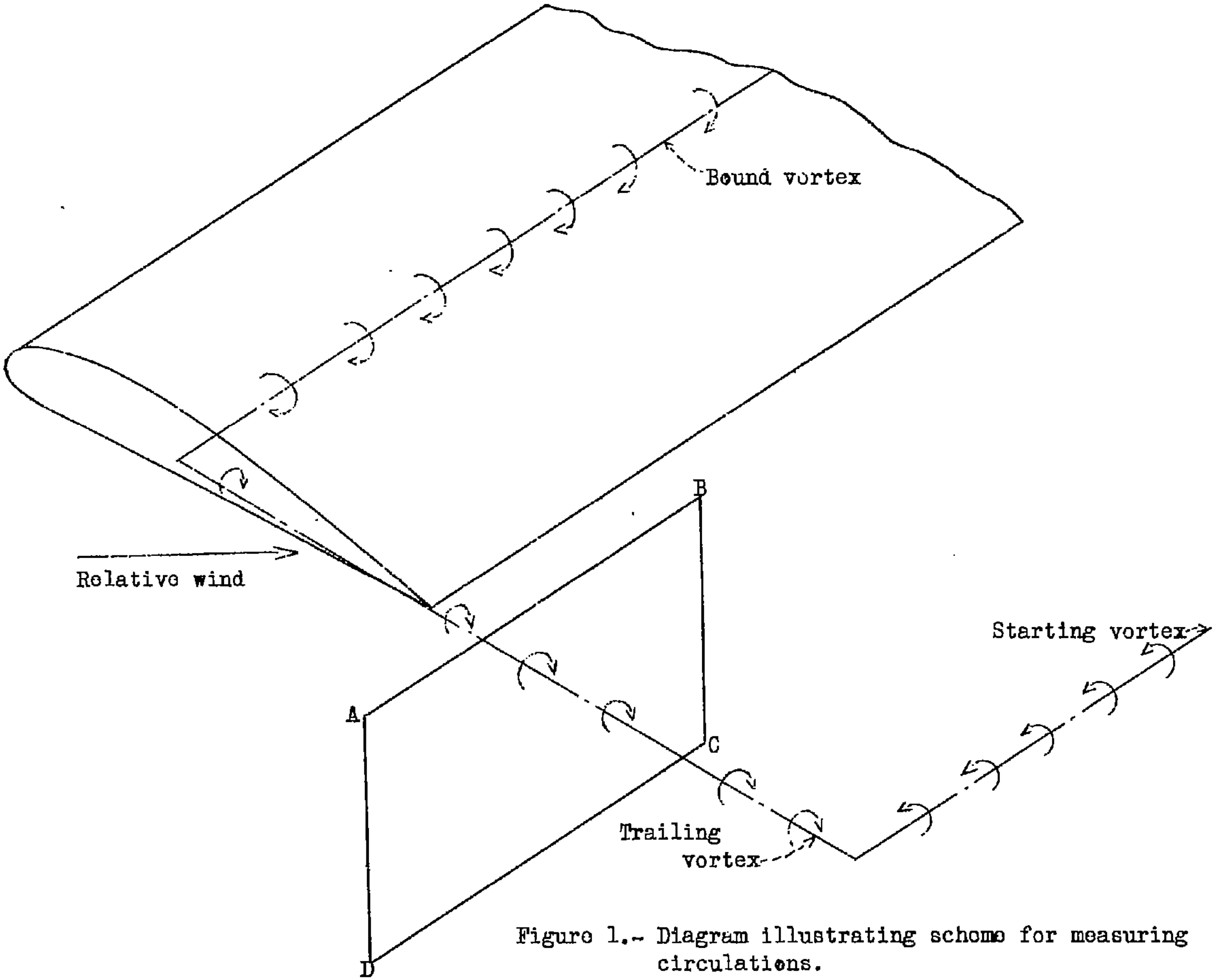


Figure 1.- Diagram illustrating scheme for measuring circulations.

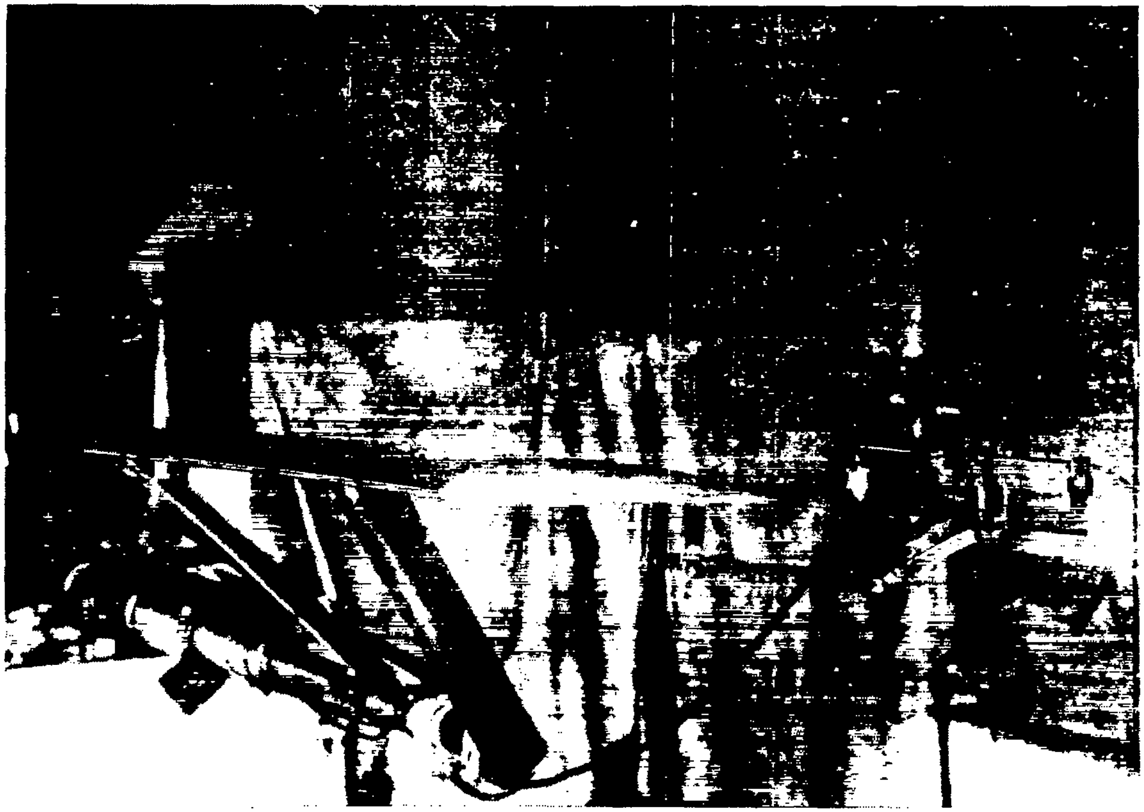
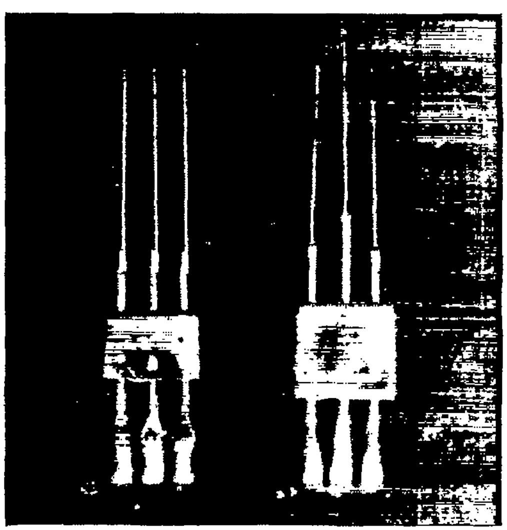


Figure 2.- Wing mounted on whirling arm with hot-wire unit projecting to the rear.



(b) (a)

(a) Direction unit
(b) Velocity difference unit

Figure 6.- Hot-wire units.

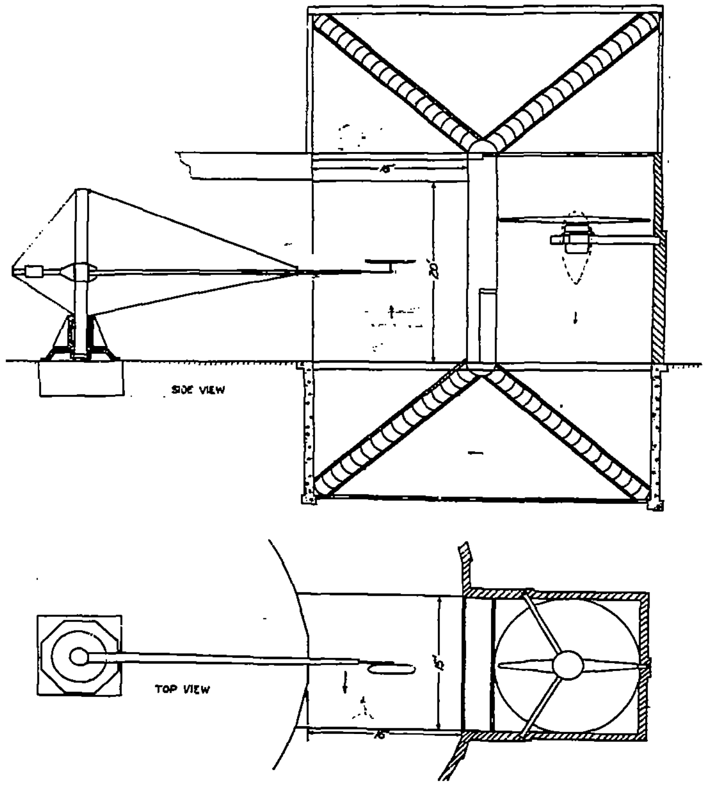


Figure 3.- Sectional views of gust tunnel and whirling arms.

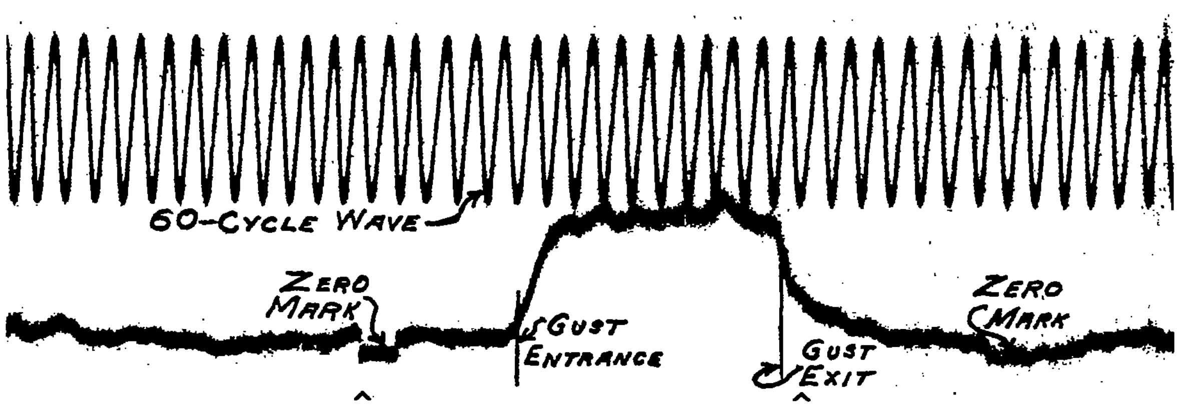


Figure 7.- Sample oscillograph record of pitch of flow behind airfoil during passage through gust.

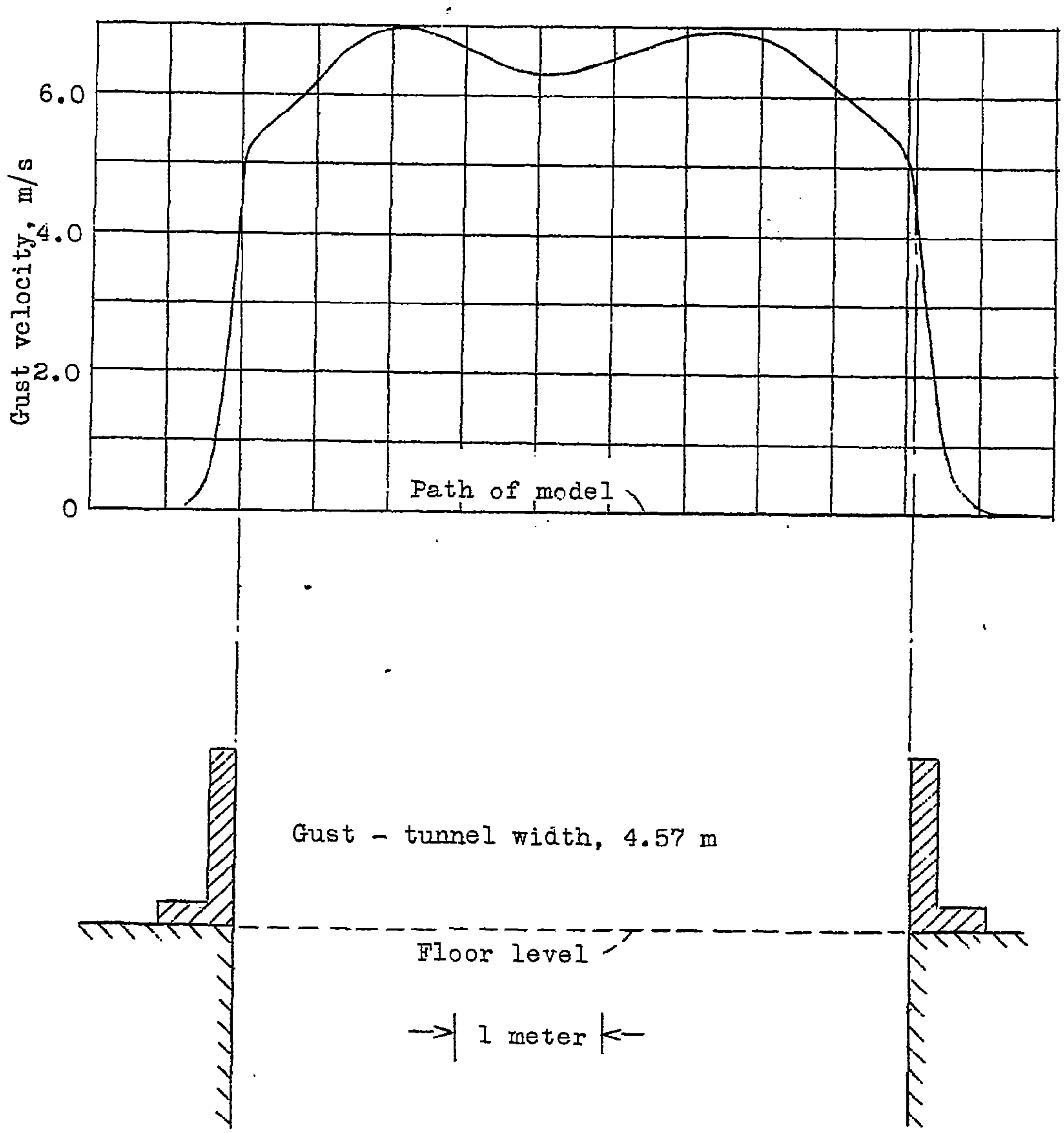


Figure 4.- Distribution of vertical-gust velocity in the path of the model.

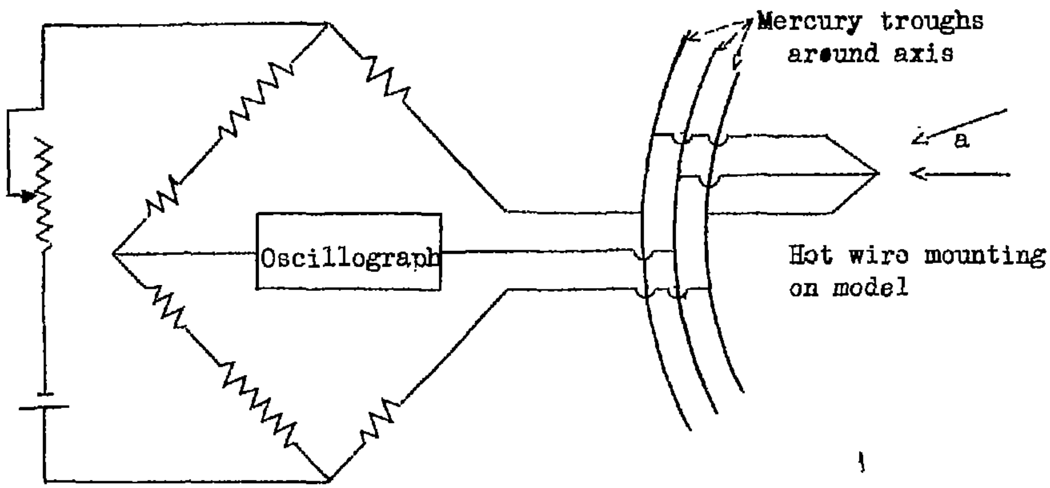


Figure 5.- Schematic wiring diagram of hot-wire apparatus.

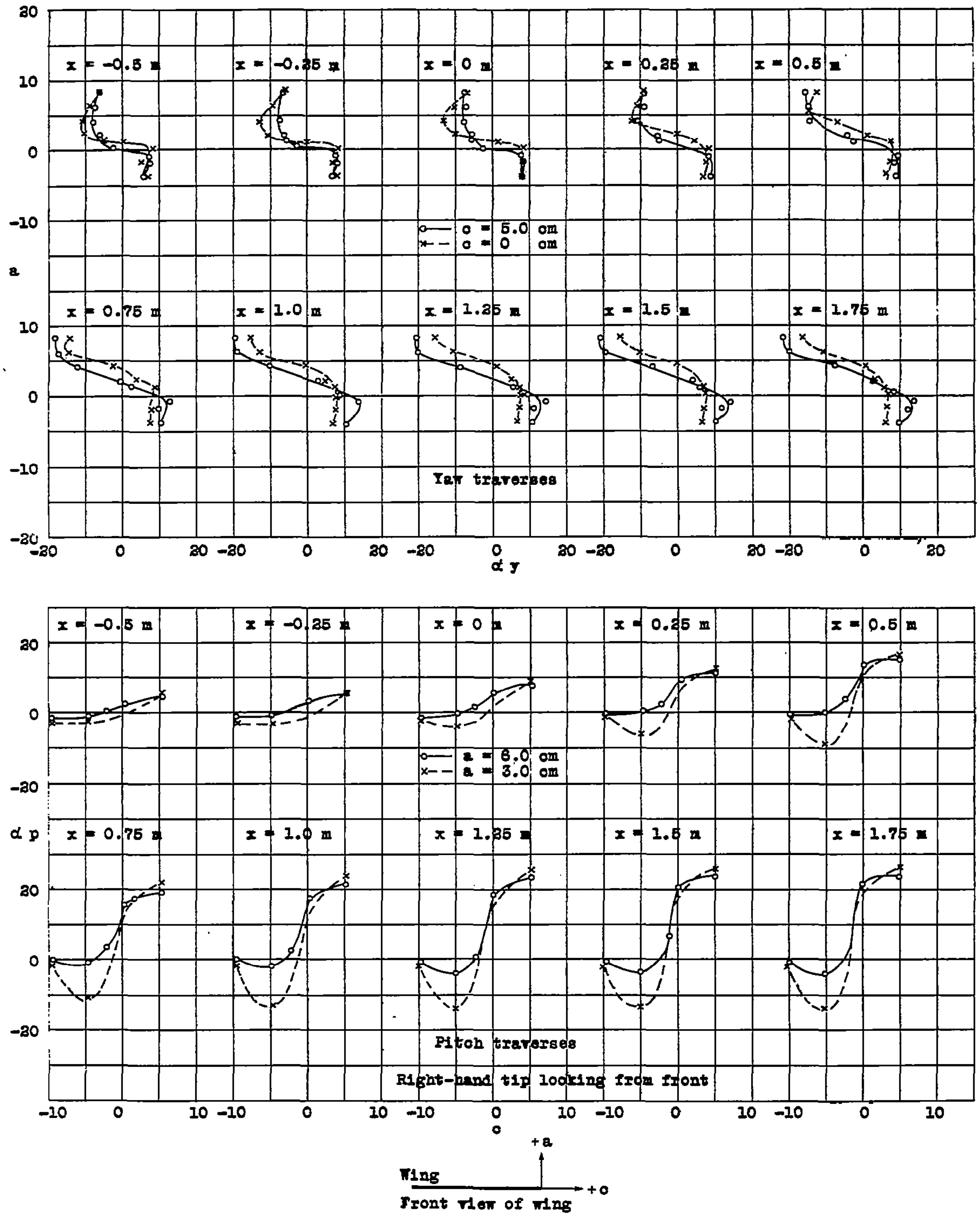


Figure 8.- Preliminary pitch and yaw measurements behind wing tip. All measurements 8cm behind trailing edge. α p , pitch angle ; α y , yaw angle.

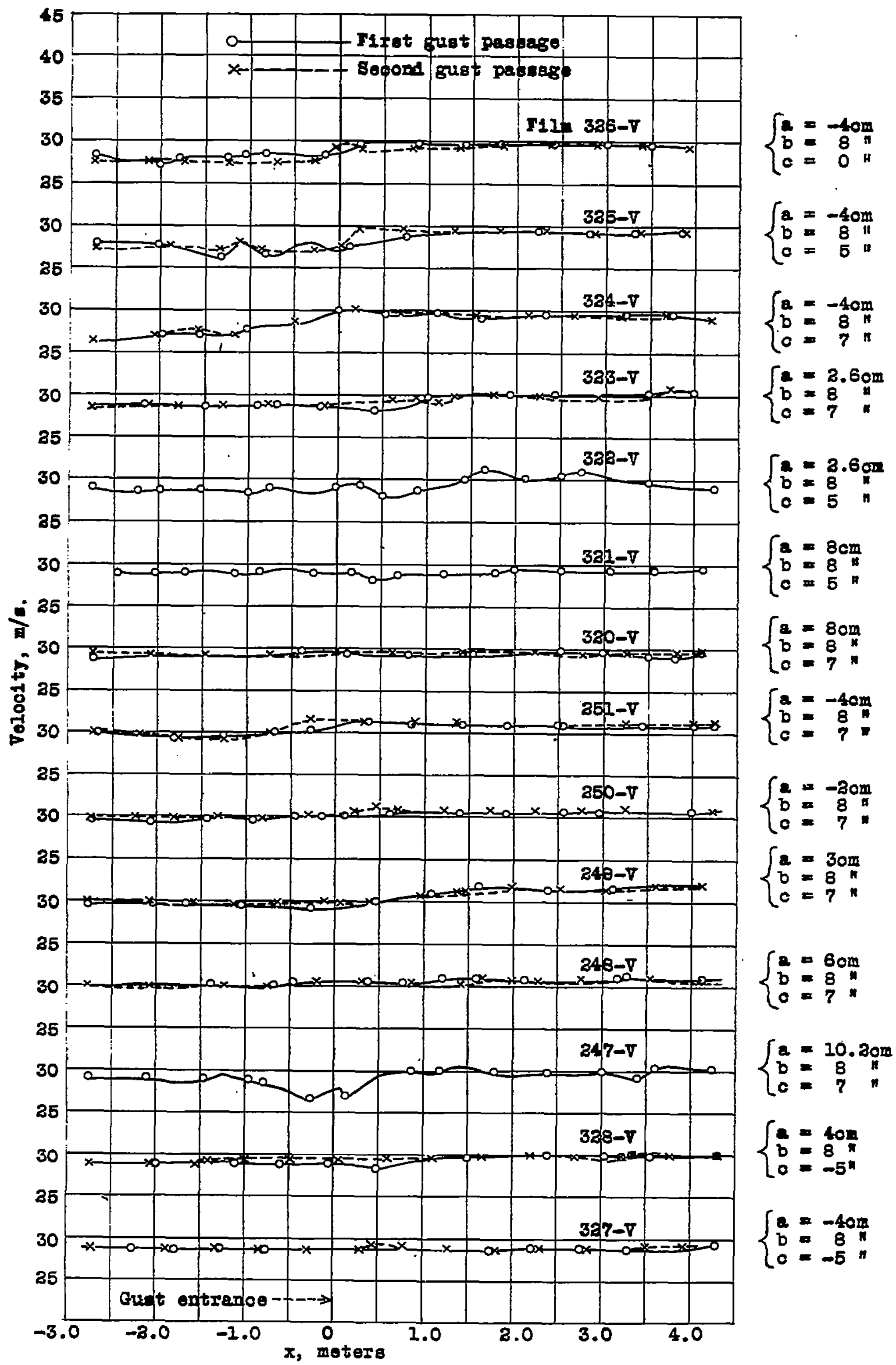
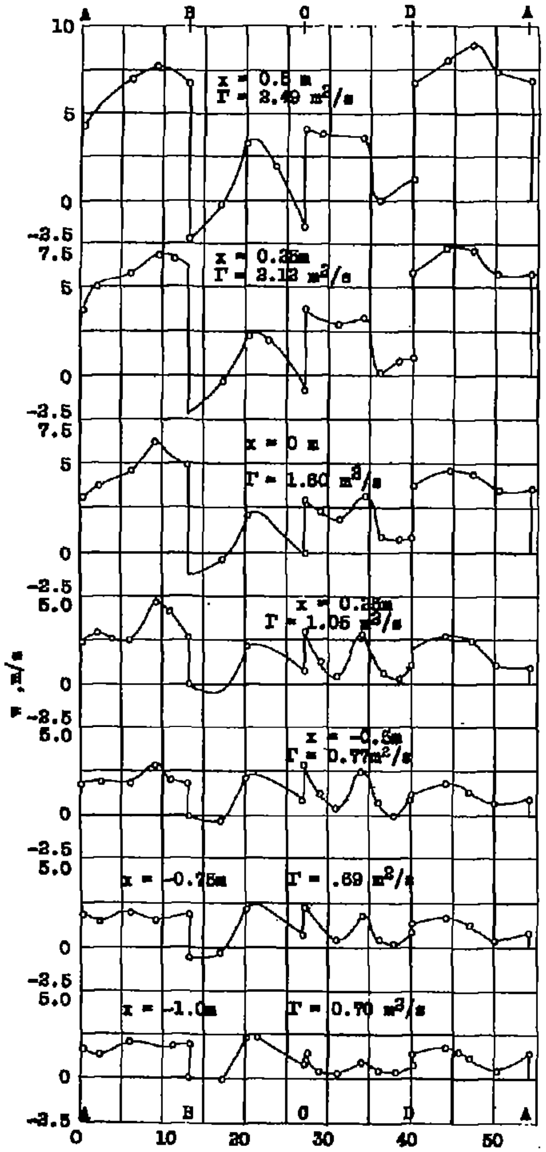
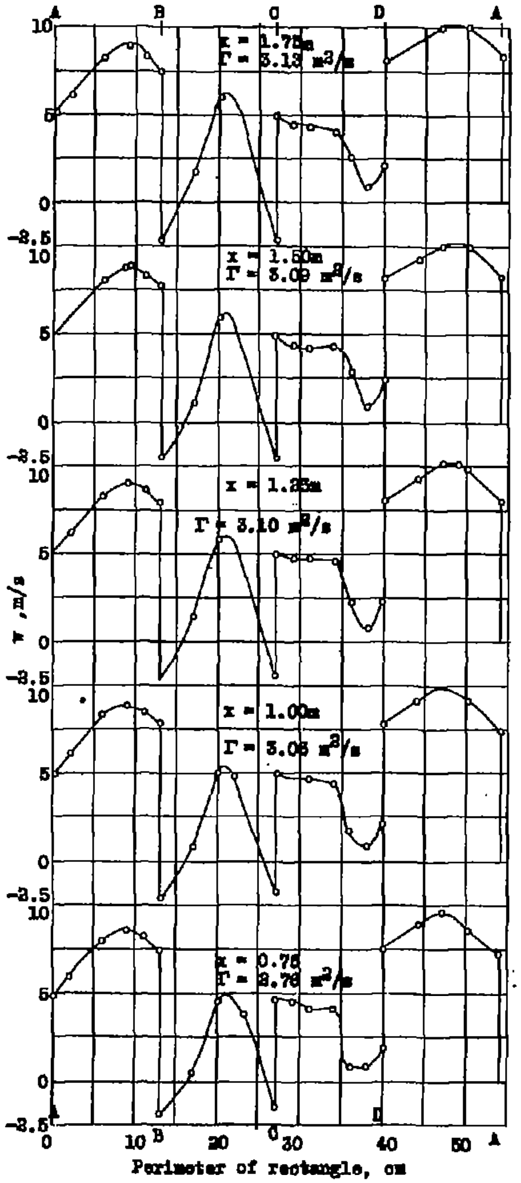


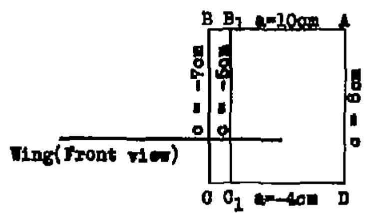
Figure 9.- Velocity measurements behind wing tip.

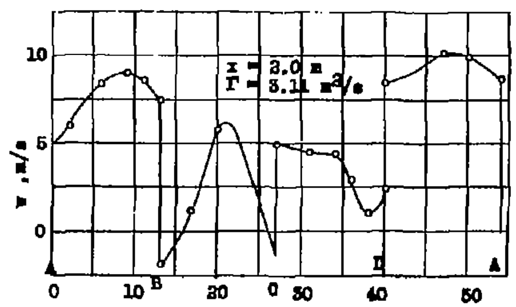




Rectangle ABCD, with dimensions as shown.

Figure 10(a).- Additional velocity(w) distributions about rectangles 8 cm behind wing tip.





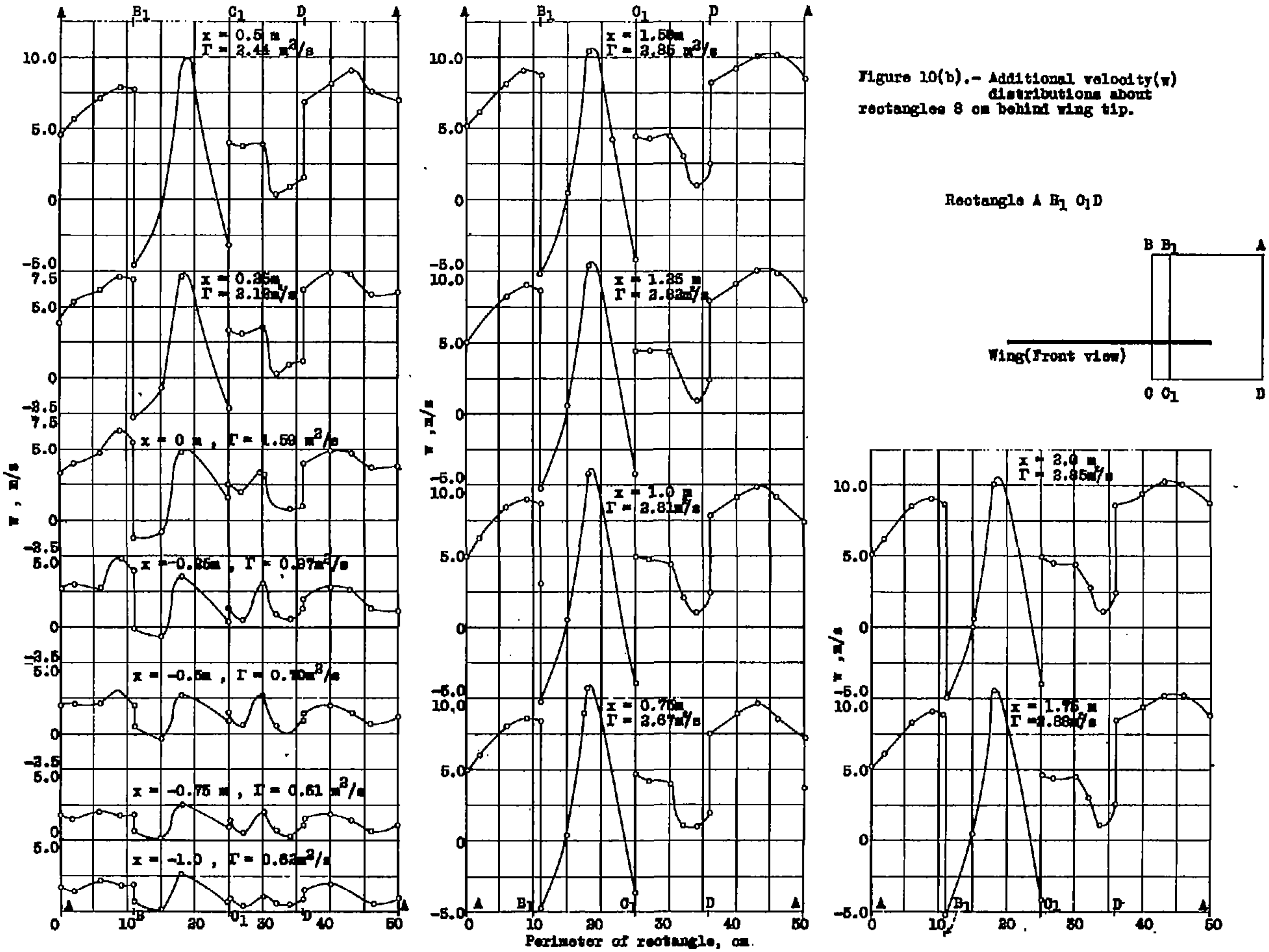


Figure 10(b).- Additional velocity(w) distributions about rectangles 8 cm behind wing tip.

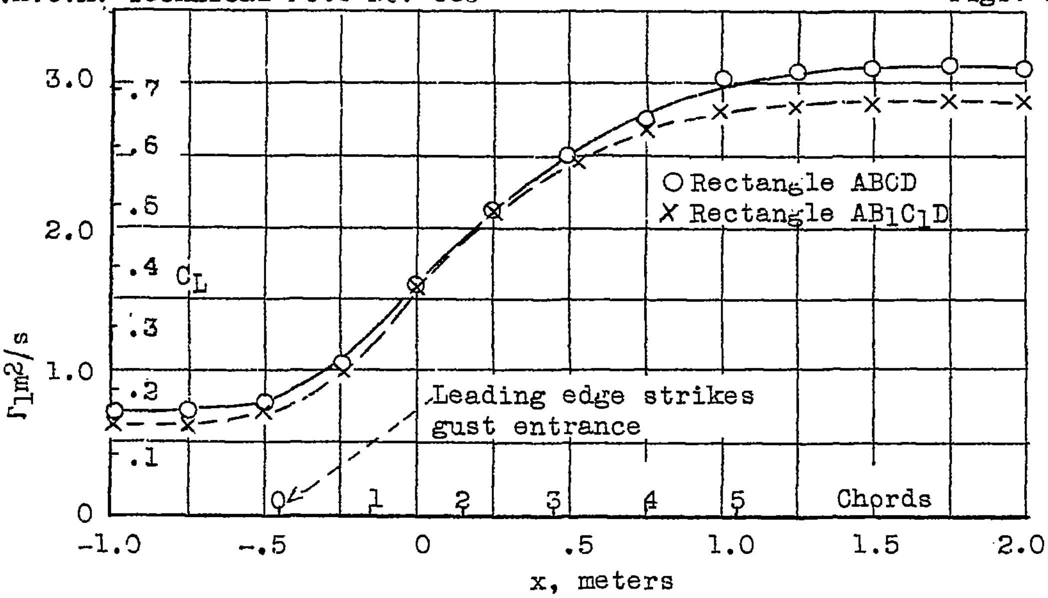


Figure 11.- Measured circulations about tip trailing vortices.

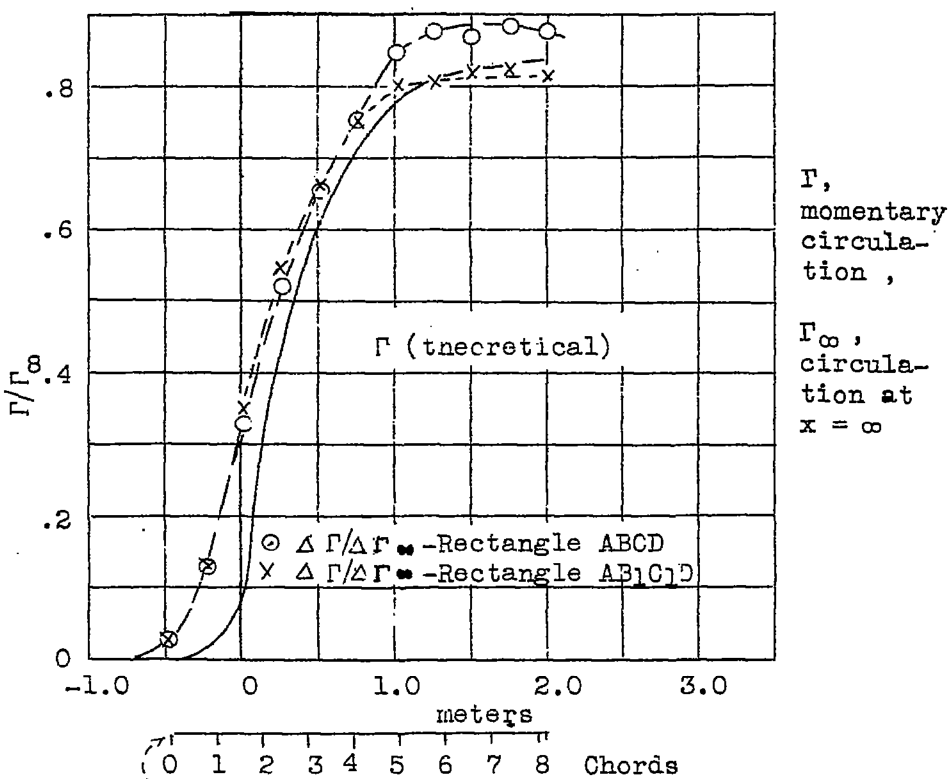


Figure 13.- Comparison of experimental circulation function with that derived from the von Kármán-Sears theory.

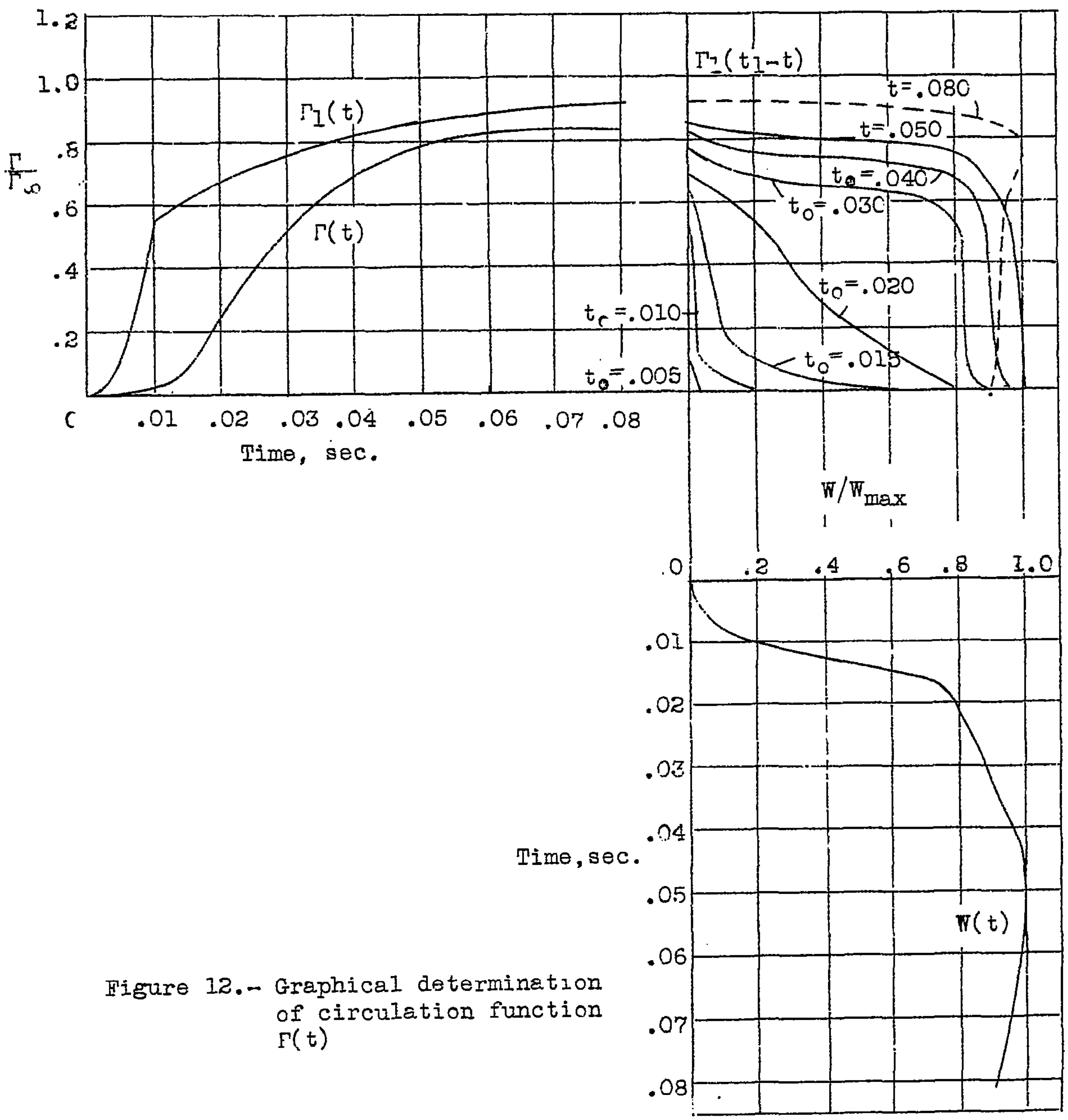


Figure 12.- Graphical determination of circulation function $\Gamma(t)$